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# Risk Sharing with Rank-dependent Utility

## 순위의존 효용하에서의 위험공유

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김 민

# Risk Sharing
# with Rank-dependent Utility

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이 논문을 경제학석사학위논문으로 제출함

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*Veritas Lux Mea*

# Risk Sharing with Rank-dependent Utility

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**Abstract**

Risk-averse agents prefer to be fully insured against the fluctuations on their endowments. One way to avoid consumption risks is to agree upon endowment transfers among them. This kind of informal insurance is what we call a risk sharing. Over an infinite time horizon, the agents can be better off through risk sharing if they are sufficiently patient.

We extend Mailath and Samuelson (2006) by allowing types of agents. A standard agent may benefit from having a pessimistic agent under rank-dependent utility as his risk sharing partner. Furthermore, it is much easier for them to have full insurance in equilibrium. We characterize a lower bound of the efficient frontier when the two agents are not patient enough. We also present our results with numerical examples.

**Keywords :** Risk sharing, Limited commitment, Repeated game with random states, Rank-dependent utility
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# Contents



# List of Figures

# Introduction

We analyze how people react to risks in their daily lives without any help from financial or legal institutions. Our interest is on risk-averse agents facing idiosyncratic endowment risks over an infinite time horizon. When there is no private information, will they cooperate in coping with endowment risks?

Consider farmers in a small village of a self-sustaining economy. To focus on idiosyncratic risks, let us suppose that there is no uncertainty on the aggregate crops in the village. Farmers would prefer to be fully insured against fluctuations on their crops every year. Without a market for insurance, what they can do best is to agree upon insuring each other by using endowment transfers. *Full insurance* (*partial insurance*) eliminates all (some) risks, so that the agents would have full (partial) intertemporal consumption smoothing. Despite the benefits from having this kind of informal insurance, there is one critical problem, a commitment problem: You cannot assure that others would help you according to the agreement. In this paper, we address two following questions: "What is the condition for the farmers to have cooperation against risks?" and "How would this cooperation look like if farmers are heterogeneous?"

In Section 1, we introduce a repeated game model of Mailath and Samuelson (2006)[1] with our new assumption. There are two infinitely-lived agents in an economy with fixed aggregate endowment. In each period, two states happen with equal probabilities. The idiosyncratic risks on the agents' endowments are perfectly negatively correlated, so that each of two states is favored by each of two agents. In state 1 (2), agent 1 (2) receives high endowment and agent 2 (1) receives low endowment.

[1] The model is described in Chapter 6 Section 3 of Mailath and Samuelson (2006). There is also a discussion about three states case in Subsection 5.

We assume that the agents are under rank-dependent utility (RDU), allowing types of the agents in the model. An agent is said to be *pessimistic* if he distorts the true probability and believes that his favorable state happens less likely. An agent under expected utility (EU) can also be categorized under RDU as a *standard* agent without probability distortions.

In Section 2, we analyze how two standard agents share risks. With sufficient patience, the agents can be fully insured against risks in equilibrium. In particular, there exists the lowest discount factor supporting full insurance in equilibrium. Given a discount factor below this critical value, the efficient equilibria of partial insurance is also characterized. In Section 3, we consider risk sharing of a standard agent with a pessimistic agent. We find an asymmetric equilibrium payoff set of full insurance and an even lower value of the lowest discount factor supporting full insurance in equilibrium. With a discount factor smaller than this value, we also characterize a lower bound of the efficient frontier of equilibria for partial insurance. The frontiers are depicted and compared with each other using numerical examples in Section 4.

**Literature review**

Our framework is originated from the mutual insurance game (Kimball, 1988; Coate and Ravallion, 1993). Kimball (1988) finds a condition on the discount factor for full insurance in equilibrium. Coate and Ravallion (1993) analyze partial insurance as well with a class of contracts about state-contingent transfers and characterize the best stationary equilibrium. The efficient frontier for partial insurance is fully characterized in a risk sharing model with two-sided limited commitment (Thomas and Worrall, 1988; Kocherlakota, 1996), a generalized model widely used in the field of macroeconomics.[2] Thomas and Worrall (1988) accounts for a long-term contract between a risk-neutral firm and a risk-averse worker in a labor market. Kocherlakota (1996) assumes two risk-averse agents to interpret a stylized fact that consumption is positively correlated with current and lagged income, conditional on per capita consumption. There is a huge research still growing along this line.

It has been suggested that households' aversion to risks comes not only from the standard feature of risk-aversion, but also from nonstandard features. Barseghyan, Molinari, O'Donoghue, and Teitelbaum (2013) show that probability distortions play a key role in explaining households' deductible choice data in both auto and home insurance. They find that people tend to overweight small probabilities of accident. Our assumption about pessimism under RDU (Quiggin, 1982) is to reflect this empirical observation. Lepetyuk and Stoltenberg (2014) also assume RDU and consider a continuum of identical agents in the model of Kocherlakota (1996). Their main interest lies on full insurance and its consumption distribution depending on the degree of pessimism and optimism. Whereas our focus is on both of the full and partial insurance equilibria payoff set for heterogeneous agents.

In the theory of insurance, it is well-known that a standard agent under EU purchases full insurance from a risk-neutral insurance company if and only if the price is actuarially fair (Mossin's Theorem). Schlesinger (1997) and Dhiab (2015) show that the price for full insurance need not be fair for a pessimistic agent. As a result, the insurance company benefits from having a pessimistic customer. This is the point from which our motivation for allowing types of agents comes. Without the market for insurance, there could be some agents who exploit others' pessimism and extract more consumption from them.

[2]See Chapter 20-21 of Ljungqvist and Sargent (2012) for discussions about the model in general with different structures of information, enforcement, and storage possibilities.

# 1 Model

There are two infinitely-lived agents and a single non-storable consumption good in the economy. The endowments are randomly determined depending on the states, while the aggregate amount is fixed as 1. Agent $i$ receives $\bar{y} \in (\frac{1}{2}, 1)$ in state $i$ for $i = 1, 2$, that is $e(1) \equiv (\bar{y}, \underline{y})$ and $e(2) \equiv (\underline{y}, \bar{y})$, where $\underline{y} \equiv 1 - \bar{y}$. Two states are equally likely. The distributions for the states are independent and identical across time periods $t \in \{0, 1, \ldots\}$.

In period $t$, after observing the endowment $e^t$, two agents simultaneously make nonnegative amount of endowment transfers to each other $\tau^t = (\tau_1^t, \tau_2^t)$. The consumption is the endowment after the transfers $c^t = (c_1^t, c_2^t)$. Both of the agents have a common utility function $u : [0, 1] \to \mathbb{R}$ and a common discount factor $\delta \in (0, 1)$. The function $u$ is differentiable, strictly increasing and strictly concave, so that the agents are risk-averse.

We assume that agent $i$ evaluates the probability using his own probability weighting function $w_i : [0, 1] \to [0, 1]$, which strictly increases and satisfies $w_i(0) = 0$ and $w_i(1) = 1$. Agent $i$ is *pessimistic* if he believes that his favorable state happens with probability $w_i(\frac{1}{2})$ which is less than $\frac{1}{2}$, while his unfavorable state is believed to happen with the complement probability $1 - w_i(\frac{1}{2})$ which is greater than $\frac{1}{2}$.[3] An agent with a linear probability weighting function is said to be *standard*.

An ex ante history in period $t$ is expressed as a sequence of the endowments and the consumption levels of the two agents in previous periods $h^t = ((e^0, c^0), \ldots, (e^{t-1}, c^{t-1}))$, which is an element of the set $\mathscr{H}^t$. Note that we replace the transfer with the consumption in histories. An ex post history in period $t$ is a combination of an ex ante history and the endowment real-

[3]Formally, an agent is said to be pessimistic if his probability weighting function is convex.

ization in that period $\tilde{h}^t = (h^t, e^t)$, which is an element of the set $\tilde{\mathscr{H}}^t$. Thus, $\mathscr{H}^0 = \{\emptyset\}$ and $\tilde{\mathscr{H}}^0 = \{e^0\}$. Let $\tilde{\mathscr{H}}$ be the set of all ex post histories. A pure strategy for agent $i$ is a function identifying the amount of transfers after each ex post history $\sigma_i : \tilde{\mathscr{H}} \to [0, 1]$. The amount of a transfer $\sigma_i(\tilde{h}^t)$ should be less than or equal to the endowment in period $t$. An ex-post history $\tilde{h}^t$ is said to be *consistent* under a strategy profile $\sigma$ if, given the implied endowment history, the transfers in each period are those specified by $\sigma$.

Agent $i$ maximizes a normalized discounted sum of payoffs given by

$$\mathbb{E}_i\left[(1-\delta)\sum_{t=0}^{\infty}\delta^t u(\sigma(\tilde{h}^t))\right],$$

where the operator $\mathbb{E}_i$ is agent $i$'s expectation with respect to the endowment distribution over histories consistent under $\sigma$.

Our equilibrium concept is pure-strategy subgame-perfect equilibria for this repeated game. Let $\mathscr{E}^p$ be the set of equilibrium payoffs and let $\mathscr{F}^*$ be the set of feasible and strictly individually rational payoffs.

## 2 Risk sharing of two standard agents

As a benchmark case, we first analyze how two standard agents share endowment risks. Note that this case corresponds to Mailath and Samuelson (2006). In this section, we summarize their results without the proofs.

Consider the stage game of risk sharing. Each agent would not give any transfers regardless of the amount of transfers from the opponent. This dominant strategy profile forms a unique Nash equilibrium, and hence a strategy profile featuring no transfers is a subgame-perfect equilibrium in the repeated game. We call this equilibrium *autarky* as there is no cooperations between the agents. The payoffs from autarky is $(\underline{v}, \underline{v})$ where

$$\underline{v} \equiv \frac{1}{2}(u(\bar{y}) + u(\underline{y})),$$

which is called the *minmax*, the payoff in the worst case in the repeated game.

Are there any subgame-perfect equilibria other than autarky? The answer is yes if the agents are patient enough (Folk theorem). In particular, we are interested in the set of equilibria called *full insurance*, where the agents are fully insured in that their consumptions are constant throughout the whole histories. We look for the condition under which full insurance equilibria exist (Subsection 2.1), then analyze equilibrium behavior when this condition fails (Subsection 2.2).

## 2.1 Full insurance

**Definition 2.1.**

1. A strategy profile $\sigma$ features *full insurance* if the corresponding consumption of the agents is not dependent on the histories, that is $c^t = (c, 1-c)$ where $c \in [0, 1]$ for all $\tilde{h}^t \in \tilde{\mathscr{H}}$.

2. A subgame-perfect equilibrium formed by $\sigma$ featuring full insurance is called *full insurance equilibrium.*

We characterize the set of full insurance equilibria in a certain way of using the most severe punishment $(\underline{v}, \underline{v})$: If one of the agents reneges on the promise at any period and state, there will be no further cooperations for insurance against risks.

Fix a full insurance strategy profile which associates with $(c, 1-c)$ for every consistent history. Let the strategy profile specifies play of autarky after any nonconsistent histories. For this strategy profile to be a subgame-perfect equilibrium, it must satisfy the incentive constraints given by

$$u(c) \geq (1-\delta)u(\bar{y}) + \delta\underline{v}, \tag{IC1}$$

$$u(1-c) \geq (1-\delta)u(\bar{y}) + \delta\underline{v}. \tag{IC2}$$

Two inequalities guarantee that the agents do not deviate from the given strategy profile when they receive high endowment. It depends on the size of the agents' discount factor whether this strategy profile can be an equilibrium.

The payoff set of full insurance equilibria, denoted by $V(\delta) \subsetneq \mathscr{E}^p$, is a subset of $\mathscr{F}^*$ such that (IC1) and (IC2) hold, that is

$$\{(u(c), u(1-c)) \in \mathscr{F}^* : u^{-1}((1-\delta)u(\bar{y})+\delta\underline{v}) \leq c \leq 1-u^{-1}((1-\delta)u(\bar{y})+\delta\underline{v})\}.$$

Since the inverse of utility function $u^{-1}$ is strictly decreasing, the set $V(\delta)$ gets bigger as $\delta$ increases. When $\delta$ goes to 1, $V(\delta)$ converges to the payoff set on the frontier within $\mathscr{F}^*$ (See Figure 1).

A natural question is that what is the lowest value of discount factor supporting full insurance in equilibrium? This value for the discount factor will be the minimum requirement for full insurance. One can find that there is a unique discount factor $\delta^* \in (0,1)$ such that the set $V$ is a singleton as $\{(\frac{1}{2}, \frac{1}{2})\}$.

Consider a full insurance strategy profile featuring symmetric consumption $(\frac{1}{2}, \frac{1}{2})$. (IC1) and (IC2) correspond to the following inequality,

$$u(\frac{1}{2}) \geq (1-\delta)u(\bar{y}) + \delta\underline{v}.$$

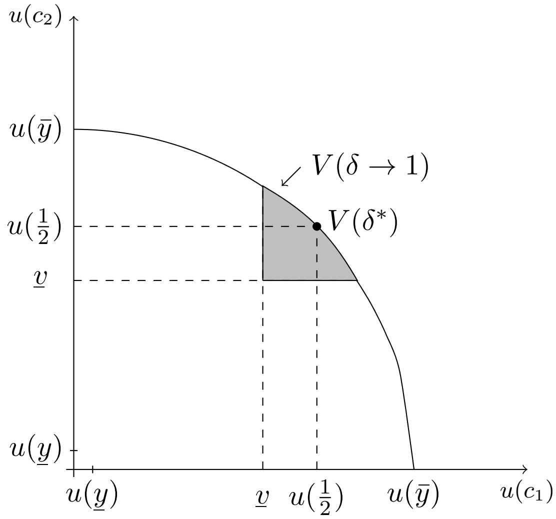



Figure 1: The set $\mathscr{F}^*$ and $V(\delta)$

Then $u(\frac{1}{2}) = (1-\delta^*)u(\bar{y}) + \delta^* \underline{v}$. Thus we have

$$\delta^* = \frac{u(\bar{y}) - u(\frac{1}{2})}{u(\bar{y}) - \underline{v}}.$$

This is the lowest discount factor that supports the full insurance equilibrium featuring consumption $(\frac{1}{2}, \frac{1}{2})$, which is the unique full insurance equilibrium featuring symmetric outcome. With $\delta$ smaller than $\delta^*$, full insurance cannot be achieved in equilibrium.

## 2.2 Partial insurance

Suppose that $\delta < \delta^*$. We would like to see whether there is an equilibrium other than autarky when $\delta$ is sufficiently large. First, we consider a class of equilibria featuring stationary outcomes in which the agents consume $(\bar{y} - \varepsilon, \underline{y} + \varepsilon)$ after any ex post history ending in endowment $e(1)$ and $(\underline{y} + \varepsilon, \bar{y} - \varepsilon)$ in endowment $e(2)$ for some $\varepsilon > 0$. Hence, this is (ex-ante) symmetric as the high-endowment agent transfers $\varepsilon$ amount to the low-endowment agent. Note that full insurance featuring symmetric outcomes belongs to this class with $\varepsilon = \varepsilon^*$, where $\varepsilon^* \equiv \bar{y} - \frac{1}{2} = \underline{y} + \frac{1}{2}$.

Since this agreement is always incentive compatible for the low-endowment agent, only the incentive constraint for the high-endowment agent matters. That is

$$(1-\delta)u(\bar{y}-\varepsilon) + \delta\frac{1}{2}[u(\bar{y}-\varepsilon) + u(\underline{y}+\varepsilon)] \geq (1-\delta)u(\bar{y}) + \delta\underline{v}, \tag{2.1}$$

or

$$\frac{1-\delta}{\delta} \leq \frac{\frac{1}{2}[u(\bar{y}-\varepsilon) + u(\underline{y}+\varepsilon)] - \underline{v}}{u(\bar{y}) - u(\bar{y}-\varepsilon)}.$$

The derivative of the right hand side of this inequality in $\varepsilon$ at $\varepsilon = \varepsilon^*$ has the same sign as $-u'(\frac{1}{2})(u(\frac{1}{2}) - \underline{v})$. Since this value is negative, reducing $\varepsilon$ below $\varepsilon^*$ increases the upper bound on $\frac{1-\delta}{\delta}$ for satisfying the incentive constraint, thereby decreasing the lower bound on values of $\delta$ for which the incentive constraint can be satisfied. This implies that there are values of the discount factor that will not support full insurance but will support stationary-outcome equilibria featuring partial insurance.

We denote the largest value of $\varepsilon$ for which (2.1) holds with equality by $\hat{\varepsilon}$ and the associated equilibrium by $\hat{\sigma}$. The collection of stationary-outcome equilibria with $\varepsilon \in [0, \hat{\varepsilon})$ is strictly dominated by $\hat{\sigma}$. Thus $\hat{\sigma}$ is the efficient, (ex ante) strongly symmetric, stationary-outcome equilibrium.

### 2.2.1 Charaterization of the efficient frontier

Consider an ex ante history that induces equilibrium consumption profiles $c(1)$ and $c(2)$ and equilibrium continuation payoff profiles $\gamma(1)$ and $\gamma(2)$ in states 1 and 2. Then,

$$(1-\delta)u(c_1(1)) + \delta\gamma_1(1) \geq (1-\delta)u(\bar{y}) + \delta\underline{v}, \tag{2.2}$$

$$(1-\delta)u(c_1(2)) + \delta\gamma_1(2) \geq (1-\delta)u(\underline{y}) + \delta\underline{v}, \tag{2.3}$$

$$(1-\delta)u(1-c_1(1)) + \delta\gamma_2(1) \geq (1-\delta)u(\underline{y}) + \delta\underline{v}, \tag{2.4}$$

$$\text{and } (1-\delta)u(1-c_1(2)) + \delta\gamma_2(2) \geq (1-\delta)u(\bar{y}) + \delta\underline{v}. \tag{2.5}$$

These incentive constraints require that each agent in each state prefer the equilibrium payoff to the punishment in autarky.

**Proposition 2.1.** *$\hat{\sigma}$ is the strongly efficient, symmetric-payoff equilibrium.*

This result allows us to identify one point on the efficient frontier.[4] Geometrically, the payoff from $\hat{\sigma}$ is at the intersection of the 45 degree line and the efficient frontier.

### 2.2.2 MAX1 and MAX2

Now we characterize other efficient equilibria as the solutions to the maximization problems called MAX1 and MAX2.

MAX1 is the following constrained maximization problem which chooses $c_1(1)$, $c_1(2)$, $\gamma(1)$ and $\gamma(2)$, to maximize the expected payoff for agent 1 given

[4] Proposition 2.1 is based on that the *average* of two equilibrium strategy profiles is also an equilibrium with better payoffs. See Lemma 6.3.1 in Mailath and Samuelson (2006) for the details.

a fixed payoff $v_2$ guaranteed for agent 2.

$$\max_{c_1(1),c_1(2),\gamma(1),\gamma(2)} \frac{1}{2}[(1-\delta)u(c_1(1)+u(c_1(2)))+\delta(\gamma_1(1)+\gamma_1(2))]$$

$$\text{subject to (2.2)-(2.5)},$$

$$\frac{1}{2}[(1-\delta)(u(1-c_1(1))+u(1-c_1(2)))+\delta(\gamma_2(1)+\gamma_2(2))] \geq v_2,$$

$$\text{and } \gamma(1),\gamma(2) \in \mathscr{E}^p.$$

The following lemma says that at least one incentive constraint must bind for this problem.

**Lemma 2.1.** *Fix $v_2$ and suppose that MAX1 has a solution in which (2.2)-(2.5) do not bind. Then*

$$c_1(1) = c_1(2),$$
$$\gamma_2(1) = \gamma_2(2) = v_2.$$

*This implies that there exists a full insurance equilibrium.*

Two lemmas below allow us to characterize all equilibria on the half of the efficient frontier using MAX1 with the guaranteed payoff for agent 2, $v_2 \in [\underline{v}_2, U_2(\hat{\sigma})]$.

**Lemma 2.2.** *Let $\sigma^*$ be a subgame-perfect equilibrium. Then agent 1 receives at least as high a payoff from an equilibrium that specifies consumption $(\bar{y}, \underline{y})$ after any ex post history in which only state 1 has been realized, and otherwise specifies equilibrium $\sigma^*$.*

**Lemma 2.3.** *The equilibrium maximizing agent 1's payoff, conditional on state 2 having been drawn in the first period, is the efficient symmetric-payoff* *stationary-outcome equilibrium $\hat{\sigma}$.*

By letting $v_2 = \underline{v}_2$, MAX1 gives an equilibrium most favorable to agent 1, denoted by $\sigma^1$. Then the payoffs will be

$$U(\sigma^1) = \frac{1}{2}U(\hat{\sigma}|e(2)) + \frac{1}{2}((1-\delta)(u(\bar{y}), u(\underline{y})) + \delta U(\sigma^1)),$$

which is a convex combination of $U(\hat{\sigma}|e(2))$ and $(u(\bar{y}), u(\underline{y}))$.

For each $v_2 \in [\underline{v}_2, U_2(\hat{\sigma})]$, there is $\varepsilon \geq 0$ such that the payoff for agent 2 equals $v_2$ in a modified equilibrium $\sigma^1$ in a way that agent 1 gives transfers of $\varepsilon$ to agent 2 in any history in which only state 1 has been realized. Let $\sigma^1_\varepsilon$ denote such class of equilibria, then the payoffs will be

$$U(\sigma^1_\varepsilon) = \frac{1}{2}U(\hat{\sigma}|e(2)) + \frac{1}{2}((1-\delta)(u(\bar{y}-\varepsilon), u(\underline{y}+\varepsilon)) + \delta U(\sigma^1_\varepsilon)).$$

In the same way, MAX2 gives another half of the efficient frontier $\sigma^2_\varepsilon$ by varying $v_1 \in [\underline{v}_1, U_1(\hat{\sigma})]$.[5]

**Proposition 2.2.** *All equilibria on the efficient frontier are characterized by MAX1 and MAX2.*

# 3 Risk sharing with a pessimistic agent

Now we consider two heterogeneous agents in evaluating the probability. Assume that agent 1 is standard, whereas agent 2 is pessimistic with a probability weighting function $w$.

The payoffs from autarky will be $(\underline{v}_1, \underline{v}_2)$, where $\underline{v}_1$ is the same as $\underline{v}$ in the

[5]See Mailath and Sameulson (2006) on page 218-219 for the detailed arguments.

previous section and

$$\underline{v}_2 \equiv w(\frac{1}{2})u(\bar{y}) + (1 - w(\frac{1}{2}))u(\underline{y}).$$

Note that $\underline{v}_1 > \underline{v}_2$, the minmax punishment is much more severe for agent 2.

### 3.1 Full insurance

We first characterize full insurance equilibrium by using the minmax, $\underline{v}_1$ and $\underline{v}_2$. Fix a full insurance strategy profile which associates with $(c, 1-c)$ for every consistent history. Let the strategy profile specifies zero transfers after any nonconsistent histories. The incentive constraints are given as

$$u(c) \geq (1-\delta)u(\bar{y}) + \delta\underline{v}_1, \tag{IC1$'$}$$

$$u(1-c) \geq (1-\delta)u(\bar{y}) + \delta\underline{v}_2. \tag{IC2$'$}$$

The payoff set for full insurance equilibria, denoted by $W(\delta) \subsetneq \mathscr{E}^p$, is a subset of $\mathscr{F}^*$ such that (IC1$'$) and (IC2$'$) hold. That is

$$\{(u(c), u(1-c)) \in \mathscr{F}^* : u^{-1}((1-\delta)u(\bar{y})+\delta\underline{v}_1) \leq c \leq 1-u^{-1}((1-\delta)u(\bar{y})+\delta\underline{v}_2)\}.$$

Note that this payoff set is asymmetric and it includes the equivalent set in Section 2 given the same discount factor, i.e. $V(\delta) \subset W(\delta)$ for all $\delta \in (0,1)$. Thus, the payoff set $W - V$ is the set of full insurance equilibrium payoffs that became available due to agent 2's pessimism. It means that there exist some equilibria where the standard agent extracts consumption from the pessimistic agent. Figure 2 illustrates the set $\mathscr{F}^*$ (the light and dark gray area). This payoff set is expanded compared to the one in Section 2 (the light gray area).

Now we look for the lowest discount factor that supports full insurance

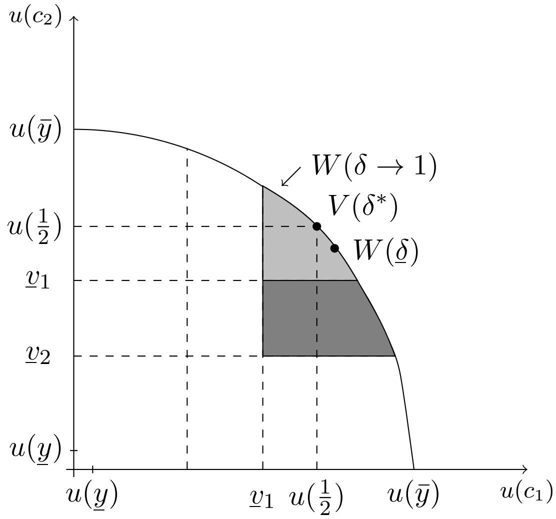



Figure 2: The set $\mathscr{F}^*$ and $W(\delta)$

equilibria. Consider a full insurance strategy profile featuring $(c, 1-c) = (\frac{1}{2}, \frac{1}{2})$ as a benchmark. For this strategy to be an equilibrium, the incentive constraints given below must be satisfied.

$$u(\frac{1}{2}) \geq (1-\delta)u(\bar{y}) + \delta \underline{v}_1, \tag{3.1}$$

$$u(\frac{1}{2}) \geq (1-\delta)u(\bar{y}) + \delta \underline{v}_2. \tag{3.2}$$

Since $\underline{v}_2 < \underline{v}_1$, the smallest discount factor that supports this strategy to be an equilibrium will be derived from (3.1), i.e. the incentive constraint for agent 1 should be binding in equilibrium. This is because agent 1 has stronger incentive for deviation to get immediate gains as his minmax is larger. Let $\delta^*$ denote the discount factor at which (3.1) is binding. Then $u(\frac{1}{2}) = (1-\delta^*)u(\bar{y}) + \delta^* \underline{v}_1$. Thus we have

$$\delta^* = \frac{u(\bar{y}) - u(\frac{1}{2})}{u(\bar{y}) - \underline{v}_1}.$$

This is the smallest discount factor that supports the full insurance equilibrium featuring $(\frac{1}{2}, \frac{1}{2})$ consumption outcome.

In contrast to Section 2, the incentive constraint for agent 2 is slack at $\delta = \delta^*$. This means that with sufficiently large $\delta < \delta^*$, we would have full insurance equilibria featuring $(\frac{1}{2} + \varepsilon, \frac{1}{2} - \varepsilon)$ for some $\varepsilon > 0$. Thus $\delta^*$ is not the lowest discount factor supporting full insurance in equilibrium, in other words, $W(\delta^*)$ is not a sigleton set. We can further reduce the value of the discount factor until both (IC1$'$) and (IC2$'$) are binding at the same time. Let us denote such discount factor by $\underline{\delta}$, then

$$u(c_1) = (1 - \underline{\delta})u(\bar{y}) + \underline{\delta}\underline{v}_1,$$
$$u(c_2) = (1 - \underline{\delta})u(\bar{y}) + \underline{\delta}\underline{v}_2,$$

and $c_1 + c_2 = 1$.

**Proposition 3.1.** *There exists a discount factor $\underline{\delta} \in (0, \delta^*)$ such that for all $\delta \geq \underline{\delta}$, there exists at least one full insurance equilibrium.*

*Proof.* To have (IC1$'$) and (IC2$'$) binding at the same time, we need to find $\delta \in (0, \delta^*)$ such that

$$c_1 = u^{-1}((1 - \delta)u(\bar{y}) + \delta\underline{v}_1),$$
$$c_2 = u^{-1}((1 - \delta)u(\bar{y}) + \delta\underline{v}_2),$$

and $c_1 + c_2 = 1$. That is

$$u^{-1}((1 - \delta)u(\bar{y}) + \delta\underline{v}_1) + u^{-1}((1 - \delta)u(\bar{y}) + \delta\underline{v}_2) = 1. \tag{3.3}$$

When $\delta = 0$, the left hand side of (3.3) equals $\bar{y} + \bar{y}$ that is greater than 1, whereas it equals $\frac{1}{2} + (1 - \delta^*)u(\bar{y}) + \delta^* \underline{v}_2$ that is less than 1 when $\delta = \delta^*$. Since $u^{-1}$ is strictly increasing and continuous in $\delta$, by the intermediate value theorem, there is a unique $\delta \in (0, \delta^*)$ satisfying (3.3). We denote this discount factor by $\underline{\delta}$. □

Therefore $W(\underline{\delta})$ is a singleton set and so there is no full insurance equilibria in which the agents' common discount factor falls short of the value of $\underline{\delta}$.

### 3.2 Partial insurance

Now we suppose $\delta < \underline{\delta}$ and follow the same approach in Section 2.2. Consider a class of equilibria featuring stationary consumption outcomes $(\bar{y} - \varepsilon, \underline{y} + \varepsilon)$ and $(\underline{y} + \varepsilon, \bar{y} - \varepsilon)$ after any ex post history ending in endowment $e(1)$ and $e(2)$, respectively, for some $\varepsilon > 0$. The incentive constraints for the high-endowment agents are

$$\begin{aligned} (1-\delta)u(\bar{y} - \varepsilon) + \delta\frac{1}{2}[u(\bar{y} - \varepsilon) + u(\underline{y} + \varepsilon)] \\ \geq (1-\delta)u(\bar{y}) + \delta\underline{v}_1 \end{aligned} \tag{3.4}$$

and

$$\begin{aligned} (1-\delta)u(\bar{y} - \varepsilon) + \delta[w(\frac{1}{2})u(\bar{y} - \varepsilon) + (1 - w(\frac{1}{2}))u(\underline{y} + \varepsilon)] \\ \geq (1-\delta)u(\bar{y}) + \delta\underline{v}_2. \end{aligned} \tag{3.5}$$

To separate $\delta$ from others, we rewrite (3.4) and (3.5) as

$$\frac{1-\delta}{\delta} \leq \frac{\frac{1}{2}[u(\bar{y} - \varepsilon) + u(\underline{y} + \varepsilon)] - \underline{v}_1}{u(\bar{y}) - u(\bar{y} - \varepsilon)}$$

and

$$\frac{1-\delta}{\delta} \le \frac{w(\frac{1}{2})u(\bar{y}-\varepsilon) + (1-w(\frac{1}{2}))u(\underline{y}+\varepsilon) - \underline{v}_2}{u(\bar{y}) - u(\bar{y}-\varepsilon)}.$$

The derivatives of the right hand sides of these two inequalities in $\varepsilon$ at $\varepsilon = \varepsilon^*$ have the same sign as $-(u(\frac{1}{2}) - \underline{v}_1)u'(\frac{1}{2})$ and $-(1-w(\frac{1}{2}))(2u(\frac{1}{2}) - u(\bar{y}) + u(\underline{y}))u'(\frac{1}{2})$, respectively. Since these values are negative, reducing $\varepsilon$ below $\varepsilon^*$ increases the upper bounds on $\frac{1-\delta}{\delta}$ for satisfying the incentive constraint, thereby decreasing the lower bound on values of $\delta$ for which the incentive constraint can be satisfied.

The largest value of $\varepsilon$ for which both of the incentive constraints hold will be derived from (3.4) as $\frac{1}{2} > w(\frac{1}{2})$. Let us denote this value by $\hat{\varepsilon}$, then

$$\begin{aligned}(1-\delta)u(\bar{y}-\hat{\varepsilon}) + \delta\frac{1}{2}[u(\bar{y}-\hat{\varepsilon}) + u(\underline{y}+\hat{\varepsilon})] \\ = (1-\delta)u(\bar{y}) + \delta\underline{v}_1.\end{aligned}$$

The associated equilibrium $\hat{\sigma}$ is the (ex ante) strongly symmetric, stationary-outcome equilibrium. The collection of equilibria with $\varepsilon \in [0, \hat{\varepsilon})$ gives payoffs that are all strictly dominated by the payoffs produced by $\hat{\sigma}$. Compared to Section 2, $\hat{\sigma}$ gives agent 1 the same payoff, but gives agent 2 smaller payoffs because of his pessimism.

Now we allow asymmetric transfers between the agents, $\varepsilon_1 \neq \varepsilon_2$, i.e. agent $i$ gives $\varepsilon_i$ of transfers to the opponent whenever he gets high endowment $\bar{y}$. In contrast to Section 2, (3.5) is slack at $\varepsilon = \hat{\varepsilon}$,

$$(1-\delta)u(\bar{y}-\hat{\varepsilon}) + \delta[w(\frac{1}{2})u(\bar{y}-\hat{\varepsilon}) + (1-w(\frac{1}{2}))u(\underline{y}+\hat{\varepsilon})] \ge (1-\delta)u(\bar{y}) + \delta\underline{v}_2.$$

This means that there could be some other stationary equilibria if we allow asymmetric transfers between the agents. The incentive constraints for a strategy profile featuring stationary outcomes with asymmetric transfers are

$$(1-\delta)u(\bar{y}-\varepsilon_1)+\delta\frac{1}{2}[u(\bar{y}-\varepsilon_1)+u(\underline{y}+\varepsilon_2)] \\ \geq (1-\delta)u(\bar{y})+\delta\underline{v}_1 \tag{3.6}$$

and

$$(1-\delta)u(\bar{y}-\varepsilon_2)+\delta[w(\frac{1}{2})u(\bar{y}-\varepsilon_2)+(1-w(\frac{1}{2}))u(\underline{y}+\varepsilon_1)] \\ \geq (1-\delta)u(\bar{y})+\delta\underline{v}_2. \tag{3.7}$$

At $\varepsilon_1=\varepsilon_2=\hat{\varepsilon}$, (3.6) holds with equality. Holding this equality, we can futher increase the value of transfers without violating (3.7), i.e. $\varepsilon_2>\varepsilon_1>\hat{\varepsilon}$. Due to strict concavity of $u$, the associated equilibrium gives higher payoffs than $\tilde{\sigma}$.

We are particularly interested in finding the largest transfers for which both (3.6) and (3.7) hold with equality. Let us define such transfers as $\hat{\varepsilon}_1,\hat{\varepsilon}_2\in(\hat{\varepsilon},\varepsilon^*)$ and refer to the associated equilibrium as $\tilde{\sigma}$. Then this equilibrium will strictly dominate any other stationary equilibria, if it exists.

**Proposition 3.2.** *$\tilde{\sigma}$ is the best stationary-outcome equilibrium.*

To find $\hat{\varepsilon}_1$ and $\hat{\varepsilon}_2$, we linearize the left hand sides of (3.6) and (3.7). By the first order Taylor approximation with respect to $\boldsymbol{\varepsilon}=(\varepsilon_1,\varepsilon_2)$ around $\boldsymbol{\varepsilon}^*=(\varepsilon^*,\varepsilon^*)$, we get the following system of linear equations.[6]

[6]The approximation error is $R_{1,\boldsymbol{\varepsilon}^*}(\boldsymbol{\varepsilon})=o(||\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^*||)$ as $\boldsymbol{\varepsilon}\to\boldsymbol{\varepsilon}^*$.

$$(1-\delta)[u(\frac{1}{2})-u'(\frac{1}{2})(\varepsilon_1-\varepsilon^*)]+\frac{\delta}{2}[2u(\frac{1}{2})-u'(\frac{1}{2})(\varepsilon_1-\varepsilon^*)$$
$$+u'(\frac{1}{2})(\varepsilon_2-\varepsilon^*)]$$
$$=(1-\delta)u(\bar{y})+\delta\underline{v}_1$$

and

$$(1-\delta)[u(\frac{1}{2})-u'(\frac{1}{2})(\varepsilon_2-\varepsilon^*)]+\delta[u(\frac{1}{2})-w(\frac{1}{2})u'(\frac{1}{2})(\varepsilon_2-\varepsilon^*)$$
$$+(1-w(\frac{1}{2}))u'(\frac{1}{2})(\varepsilon_1-\varepsilon^*))]$$
$$=(1-\delta)u(\bar{y})+\delta\underline{v}_2.$$

In matrix form, they are $A(\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^*)=\boldsymbol{y}$ where

$$A=\begin{bmatrix}-u'(\frac{1}{2})(1-\frac{\delta}{2}) & u'(\frac{1}{2})\frac{\delta}{2}\\ u'(\frac{1}{2})\delta(1-w(\frac{1}{2})) & -u'(\frac{1}{2})(1-\delta+\delta w(\frac{1}{2}))\end{bmatrix}$$
$$\text{and}\quad \boldsymbol{y}=\begin{bmatrix}(1-\delta)u(\bar{y})-u(\frac{1}{2})+\delta\underline{v}_1\\ (1-\delta)u(\bar{y})-u(\frac{1}{2})+\delta\underline{v}_2\end{bmatrix}.$$

Note that the determinant of matrix $A$, $\det(A)=(u'(\frac{1}{2}))^2(1-\delta-\frac{\delta}{2}+\delta w(\frac{1}{2}))$, equals zero if $\delta=1/(1+\frac{1}{2}-w(\frac{1}{2}))$. Thus matrix $A$ is invertible with positive determinant[7] if

$$\delta<\frac{1}{1+\frac{1}{2}-w(\frac{1}{2})}. \tag{3.8}$$

This condition on the discount factor can be rewritten as $\frac{1}{2}-w(\frac{1}{2})<\frac{1-\delta}{\delta}$.

[7]With $\det(A)>0$, $A^{-1}$ has negative elements, so that $\hat{\varepsilon}_1,\hat{\varepsilon}_2<\varepsilon^*$.

Let $r$ denote the discount rate, i.e. $\delta \equiv \frac{1}{1+r}$. Then (3.8) becomes

$$\frac{1}{2} - w(\frac{1}{2}) < r,$$

which requires the given discount rate to be greater than the degree of agent 2's pessimism, measured by the difference between the true and perceived probabilities, $\frac{1}{2} - w(\frac{1}{2})$.

Therefore, for a sufficiently large discount factor such that (3.8) holds,[8] we get the largest transfers

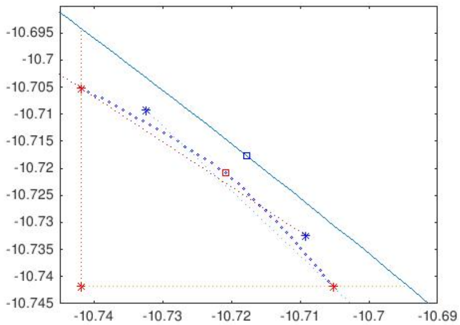
$$\begin{bmatrix} \hat{\varepsilon}_1 \\ \hat{\varepsilon}_2 \end{bmatrix} \equiv A^{-1}\boldsymbol{y} + \boldsymbol{\varepsilon}^*.$$

### 3.2.1 Charaterization of a lower bound for the efficient frontier

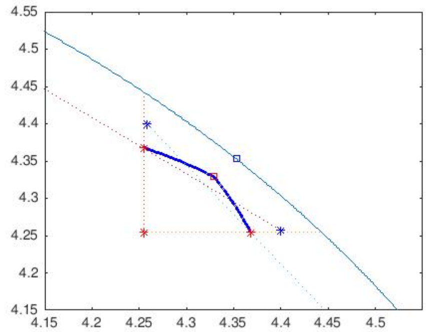
By applying the same logic as in Section 2, we can find an analogous set of equilibria $\tilde{\sigma}^1_\varepsilon$ which gives payoffs as convex combinations of $U(\tilde{\sigma}|e(2))$ and $(u(\bar{y} - \varepsilon), u(\underline{y} + \varepsilon))$ for some $\varepsilon \geq 0$. In a similar way, we get another set of equilibria $\tilde{\sigma}^2_\varepsilon$. However, we cannot guarantee that the equilibrium $\tilde{\sigma}$ is efficient. Thus our characterization gives a lower bound of the efficient frontier.

**Proposition 3.3.** *The sets of equilibria, $\tilde{\sigma}^1_\varepsilon$ and $\tilde{\sigma}^2_\varepsilon$, form a lower bound of the efficient frontier.*

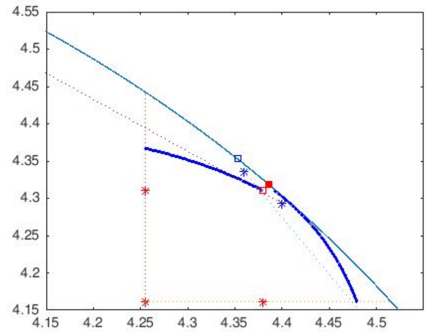
[8] The condition (3.8) is not necessary to have the best stationary equilibrium. This condition is redundant if $\underline{\delta} < \frac{1}{1+\frac{1}{2}-w(\frac{1}{2})}$. In that case, there always exist $\hat{\varepsilon}_1$ and $\hat{\varepsilon}_2$ for which the incentive constraints, (3.6) and (3.7), hold with equality. If $\delta \geq \frac{1}{1+\frac{1}{2}-w(\frac{1}{2})}$, one can find the largest possible transfers that respect the incentive constraints. Then the associated equilibrium will also strictly dominate $\hat{\sigma}$.




(a) Ljungqvist and Sargent (2012)




(b) Two standard agents




(c) With a pessimistic agent

Figure 3: The payoffs of $\sigma_\varepsilon^1$ and $\sigma_\varepsilon^2$

# 4 Numerical examples

Assume that the instantaneous utility function is $u(c) = \frac{c^{1-\rho}}{1-\rho}$ with a constant relative risk aversion, $\rho > 0$ and $\rho \neq 1$. In the following, we show some numerical examples of our results using `MATLAB`.

First we can replicate the *two-state example* of Ljungqvist and Sargent (2012)[9] by setting $\rho = 1.1$ and $\bar{y} = 0.6$ as Figure 3 (a) illustrates. The symmetric full insurance equilibrium featuring $(\frac{1}{2}, \frac{1}{2})$ consumption (blue square) is supported by $\delta^* = 0.889$. By setting $\delta = 0.85$, we find the transfers

[9]See Chapter 21 Section 10 in Ljungqvist and Sargent (2012).

$\hat{\varepsilon} = 0.064$. In equilibrium $\hat{\sigma}$, the corresponding consumption is 0.536 in high-endowment state, 0.464 in low-endowment state. Thus the equilibrium payoffs are $U(\hat{\sigma}) = (-10.7208, -10.7208)$ (red square). The efficient frontier is described by small blue dots. Two blue stars describe the continuation payoffs after state 1 and 2 realize, respectively, that is $U(\hat{\sigma}|e(j))$ for $j = 1, 2$.

In the following, we set $\rho = 0.8$, $\bar{y} = 0.75$ and $\delta = 0.688$ and compare the two results in Section 2 and 3 as Figure 3 (b) and (c) illustrate.

**Two standard agents**. We get $\delta^* = 0.79$ and $\hat{\varepsilon} = 0.121$. In equilibrium $\hat{\sigma}$. the payoffs are $U(\hat{\sigma}) = (4.3286, 4.3286)$ (red square). The efficient frontier is described by the thick blue line.

**With a pessimistic agent**. Here we have one additional parameter $w(\frac{1}{2})$ being set to 0.4. Then the lowest discount factor is $\underline{\delta} = 0.719$ at which there is one full insurance equilibrium (red filled square). The transfers are $\hat{\varepsilon}_1 = 0.1979$, and $\hat{\varepsilon}_2 = 0.2297$. In equilibrium $\tilde{\sigma}$, $U(\tilde{\sigma}) = (4.3796, 4.3106)$ (red square). The lower bound of the efficient frontier is described by the thick blue lines.

In comparison with panel (b), we can see that the frontier in panel (c) is expanded and skewed to agent 1. The standard agent receives more payoffs with a pessimistic agent than with another standard one in risk sharing.

## Concluding remarks

In this paper, we extended a repeated game model of risk sharing suggested in the literature. Under our framework with RDU, we saw that it is beneficial to have a pessimistic risk sharing partner for a standard agent. In their risk sharing, the condition for existence of full insurance equilibria was easier to be satisfied. The equilibrium payoff set for full insurance was expanded and skewed to the standard agent. In numerical examples, we found that the equilibrium payoff sets for partial insurance also became favorable to the standard agent.

We believe that our results may help in answering how and why the insurance market and insurance companies had emerged. Every person is risk-averse, but some could be much more pessimistic about the future. Thus those relatively optimistic people would have the opportunity to exploit others' pessimism and make a profit.

For future research, we suggest to find the efficient frontier for partial insurance in Section 3. After that, one could directly compare two efficient frontiers in Section 2 and 3. Comparative statics depending on the degree of pessimism could also be done as a simple exercise. One could also further generalize the model by allowing $k$ states and $n$ agents. Finally, it would be interesting to compare the risk sharings of two pessimistic agents and two standard agents.

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# Appendix

Here we provide `MATLAB` code for the numerical examples in Section 4.

```
%CRRA utility function w/ parameter rho
function result=u1(x,rho)
result=(x.^(1-rho))/(1-rho);
end

%Agent 2 utility when agent 1 gets x
function result=u2(x,rho)
result=((1-x).^(1-rho))/(1-rho);
end

%inverse CRRA utility function w/ parameter rho
function result=u_inv(x,rho)
result=(x*(1-rho)).^(1/(1-rho));
end

%derivative of CRRA utility function w/ parameter rho
function result=u1d(x,rho)
result=x.^(-rho);
end
```

**Two standard agents.**

```
rho=0.8; % degree of relative risk aversion
y_bar=0.75; %high endowment
y_ubar=1-y_bar; %low endowment
v_ubar = 0.5*(u1(y_bar,rho)+u1(y_ubar,rho)); %minmax
delta=0.688; %delta<delta_star

%consumptions
c = 0:0.001:1;
%discount factors
d = 0:0.001:1;

%consumption that gives minmax
[difference1, index1]=min(abs(v_ubar-u1(c,rho)));
c_v=c(index1);

%finding delta_star
[difference2, index2]=min(abs(u3(y_bar,d,rho,v_ubar)-u1(.5,rho)));
d_star=d(index2);

%finding epshat
Ui_i=(1-delta)*u1(y_bar,rho)+delta*v_ubar;
c1=c(c <y_bar); %esphat is less than espstar and greater than 0
[difference3, index3]=min(abs(Ui_i-U(c1,delta,rho)));
epshat=y_bar-c(index3);

```

```
%payoffs at sigma hat
Uj_i=(1-delta)*u1(y_ubar+epshat,rho)+0.5*delta*(u1(y_bar-epshat,rho)+u1
    (y_ubar+epshat,rho));
Ui_sighat=0.5*(Ui_i+Uj_i);

%payoffs at sigma 1 and 2
U1_sigma1=1/(2-delta)*(Uj_i+(1-delta)*u1(y_bar,rho));
U2_sigma2=U1_sigma1;

%frontier
 v_2=v_ubar:0.001:Ui_sighat;
 e=zeros(size(v_2));
 x=zeros(size(v_2));
for  i=1:numel(v_2)

[difference4, index4]=min(abs(v_2(i)-U2_1(c1,delta,rho,Ui_i)));
e(i)=c(index4)-y_ubar; %epsilons
x(i)=U2_1(y_bar-e(i),delta,rho,Uj_i);
end

%plot
plot(u1(c,rho), u2(c,rho)); %utility possibility frontier
axis([4.15 4.55 4.15 4.55]);

hold on;
plot(u1(.5,rho),u1(.5,rho),'bs'); %full insurance 1/2
plot(v_ubar,v_ubar,'r*'); %minmax
plot([v_ubar,v_ubar],[v_ubar,u2(c_v,rho)],':',[v_ubar,u1(1-c_v,rho)],[
    v_ubar,v_ubar],':'); %F* set

scatter(x,v_2,0.6,'blue'); %frontier
scatter(v_2,x,0.6,'blue');%frontier

plot(Ui_sighat,Ui_sighat,'rs'); %sigma hat eqbm
plot(Ui_i,Uj_i,'b*'); %sigma continuation state1
plot(Uj_i,Ui_i,'b*'); %sigma continuation state2
plot(u1(y_bar,rho),u2(y_bar,rho),'rs'); %(u(ybar),u(yubar))
plot(u1(y_ubar,rho),u2(y_ubar,rho),'rs');%(u(yubar),u(ybar))
plot([Uj_i,u1(y_bar,rho)],[Ui_i,u2(y_bar,rho)],':',[u1(y_ubar,rho),Ui_i
    ],[u2(y_ubar,rho),Uj_i],':'); %initial line seg
plot(U1_sigma1,v_ubar,'r*'); %payoffs in sigma 1
plot(v_ubar,U2_sigma2,'r*'); %payoffs in sigma 2
```

### With a pessimistic agent.

```
rho=0.8; % degree of relative risk aversion
y_bar=0.75; %high endowment
y_ubar=1-y_bar; %low endowment
w=0.4; %probability distortion
v_ubar1 = 0.5*(u1(y_bar,rho)+u2(y_bar,rho)); %minmax 1
v_ubar2 = w*u1(y_bar,rho)+(1-w)*u2(y_bar,rho); %minmax 2
eps_star=y_bar-.5;
delta=0.688; %delta<d_ubar

%consumptions
```

```
c = 0:0.0001:1;
%discount factors
d = 0:0.0001:1;

%consumption that gives minmax
[difference1, index1]=min(abs(v_ubar1-u1(c,rho)));
[difference2, index2]=min(abs(v_ubar2-u1(c,rho)));
c_v1=c(index1);
c_v2=c(index2);

%finding delta_ubar
[difference3, index3]=min(abs(u_inv((1-d)*u1(y_bar,rho)+d*v_ubar1,rho)+u_inv((1-d)*u1(y_bar,rho)+d*v_ubar2,rho)-1));
d_ubar=d(index3);
[difference4, index6]=min(abs((1-d_ubar)*u1(y_bar,rho)+d_ubar*v_ubar1-u1(c,rho)));
[difference5, index5]=min(abs((1-d_ubar)*u1(y_bar,rho)+d_ubar*v_ubar2-u1(c,rho)));
c_f1=c(index6);
c_f2=c(index5);

v_bar1=(1-delta)*u1(y_bar,rho)+delta*v_ubar1;
v_bar2=(1-delta)*u1(y_bar,rho)+delta*v_ubar2;

%finding epshat_1,epshat_2 by first order Taylor expansion
A=[-u1d(.5,rho)*(1-delta*.5) u1d(.5,rho)*delta*.5 ; u1d(.5,rho)*delta*(1-w) -u1d(.5,rho)*(1-delta+delta*w)];
y=[(1-delta)*u1(y_bar,rho)-u1(.5,rho)+delta*v_ubar1 ; (1-delta)*u1(y_bar,rho)-u1(.5,rho)+delta*v_ubar2];
epshat=inv(A)*y+[eps_star;eps_star];

%%At sigma hat
%current and continuation in good state
U1__1=(1-delta)*u1(y_bar,rho)+delta*v_ubar1;
U2__2=(1-delta)*u1(y_bar,rho)+delta*v_ubar2;

%current and continuation in bad state
U2__1=(1-delta)*u1(y_ubar+epshat(1),rho)+delta*(w*u1(y_bar-epshat(2),rho)+(1-w)*u1(y_ubar+epshat(1),rho));
U1__2=(1-delta)*u1(y_ubar+epshat(2),rho)+0.5*delta*(u1(y_bar-epshat(1),rho)+u1(y_ubar+epshat(2),rho));

U_sighat=[.5*(U1__1+U1__2); w*U2__2+(1-w)*U2__1];
c1=c(c <y_bar);

%frontier
 v_2=v_ubar2:0.001:U_sighat(2);
 v_1=v_ubar1:0.001:U_sighat(1);
 e1=zeros(size(v_1));
 x1=zeros(size(v_1));
 e2=zeros(size(v_2));
 x2=zeros(size(v_2));

for  i=1:numel(v_1) %MAX2
[difference6, index6]=min(abs(v_1(i)-U2_1(c1,delta,rho,U1__1)));
e1(i)=c(index6)-y_ubar; %epsilons
x1(i)=U2_11(y_bar-e1(i),delta,rho,U2__1,w); %agent 2 gets
```

```
end

for  i=1:numel(v_2) %MAX1
[difference7, index7]=min(abs(v_2(i)-U2_11(c1,delta,rho,U2__2,w)));
e2(i)=c(index7)-y_ubar; %epsilons
x2(i)=U2_1(y_bar-e2(i),delta,rho,U1__2); %agent 1 gets
end

%plot
plot(u1(c,rho), u2(c,rho)); %utility possibility frontier
axis([4.15 4.55 4.15 4.55]);

hold on;
plot(u1(.5,rho),u1(.5,rho),'bs'); %full insurance 1/2
plot(u1(c_f1,rho),u1(c_f2,rho), 'rs', 'MarkerFaceColor','r'); %full insurance with d_ubar

plot(v_ubar1,v_ubar2,'r*'); %minmax
plot([v_ubar1,v_ubar1],[v_ubar2,u2(c_v1,rho)],':',[v_ubar1,u1(1-c_v2,rho)],[v_ubar2,v_ubar2],':'); %F* set

scatter(x2,v_2,0.6,'blue'); %frontier
scatter(v_1,x1,0.6,'blue');%frontier

plot(U_sighat(1),U_sighat(2),'rs'); %sigma hat eqbm
plot(U1__1,U2__1,'b*'); %sigma continuation state1
plot(U1__2,U2__2,'b*'); %sigma continuation state2
plot(u1(y_bar,rho),u2(y_bar,rho),'rs'); %(u(ybar),u(yubar))
plot(u1(y_ubar,rho),u2(y_ubar,rho),'rs');%(u(yubar),u(ybar))
plot([U1__2,u1(y_bar,rho)],[U2__2,u2(y_bar,rho)],':',[u1(y_ubar,rho),U1__1],[u2(y_ubar,rho),U2__1],':'); %initial line seg
plot(U_sighat(1),v_ubar2,'r*'); %payoffs in sigma 1
plot(v_ubar1,U_sighat(2),'r*'); %payoffs in sigma 2
```

# 초 록

본 연구는 위험기피 행위자들이 소비 부존자원에 대한 변동을 어떻게 대처하는지 살펴본다. 서로에 대한 부존자원이전에 관한 합의가 있다면, 이들은 소비위험 전부로부터 보호받을 수 있다. 이러한 비형식적인 보험을 일컬어 위험공유라고 한다. 충분한 참을성이 있다면, 무한기에서의 행위자들은 위험공유로 더 큰 효용을 누릴 수 있다.

순위의존 효용을 도입하고 서로 다른 행위자 타입을 상정함으로써, 본 연구는 Mailath and Samuelson (2006)의 모형을 확장한다. 표준적인 행위자는 비관적인 행위자와 위험을 공유하여 이득을 볼 수 있다. 또한, 이 경우에 전부보험이 균형에서 비교적 쉽게 달성될 수 있다. 본 연구는 이어 행위자들이 충분히 참을성 있지 않을 때의 효율적 균형보수 곡선의 하계를 묘사하고, 마지막으로 수치 예를 통해 결과를 나타낸다.

**주요어 :** 위험공유, 제한된 약속, 랜덤 상태에서의 반복 게임, 순위의존 효용
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